Instructor's Resource Guide

Calculus: Early Transcendental Functions

SEVENTH EDITION

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Chapter 1 Preparation for Calculus

Chapter Comments

Chapter 1 is a review chapter and, therefore, should be covered quickly. Spend about 5 or 6 days on this chapter, placing most of the emphasis on Section 1.3. Of course, you cannot cover every single item that is in this chapter, so this is a good opportunity to encourage your students to read the book. To convince your students of this, assign homework problems or give a quiz on some of the material that you do not go over in class. The tools in this chapter need to be readily at hand. That is, they need to be memorized.

Sections 1.1 and 1.2 can be covered in a day. Students at this level of mathematics have graphed equations before, so let them read about that information on their own. Discuss intercepts, emphasizing that they are points, not numbers, and should be written as ordered pairs. Also discuss symmetry with respect to the x-axis, the y-axis, and the origin. Be sure to do a problem like Example 5 in Section 1.1. Students need to be able to find the points of intersection of graphs in order to calculate the area between two curves in Chapter 7.

In Section 1.2, discuss the slope of a line, the point-slope form of a line, equations of vertical and horizontal lines, and the slopes of parallel and perpendicular lines. Students need to know the point-slope form of a line because this is needed to write the equation of a tangent line in Chapter 2.

Students need to know everything in Section 1.3, so carefully go over the definition of a function, domain and range, function notation, transformations, the terms algebraic and transcendental, and the composition of functions. Because students need practice handling Δx, be sure to do an example calculating $f(x + \Delta x)$. Your students should know the graphs of the six basic functions in Figure 1.27. A knowledge of even and odd functions will be helpful with definite integrals.

Section 1.4 provides students with a comprehensive review of the fundamentals of trigonometry. The parts of an angle and degree and radian measures are discussed. The section presents the six trigonometric functions along with their domains, ranges, how to find values, and graphing, and develops a list of important properties. Uses of right triangles, circles, and the standard unit circle are presented.

Section 1.5 reviews inverse functions, a topic that which many students struggle. It is worthwhile to spend time on Figure 1.48, reviewing both notation and domains and ranges. It is important to stress the requirement of a function being one-to-one in order to have an inverse function (see Example 2), as you lead into a discussion of the inverse trigonometric functions and their restrictions. Example 5 can lead to some good discussions on angles versus values and exact values.

Section 1.6 begins with the exponential function and finishes with the logarithmic function. As this is a review of these functions, you may find it helpful to begin with Figure 1.62, reminding students that these two functions are inverse functions of each other.

The authors assume students have a working knowledge of inequalities, the formula for the distance between two points, absolute value, and so forth. If needed, you can find a review of these concepts in Appendix C.

Section 1.1 Graphs and Models

Section Comments

1.1 **Graphs and Models**—Sketch the graph of an equation. Find the intercepts of a graph. Test a graph for symmetry with respect to an axis and the origin. Find the points of intersection of two graphs. Interpret mathematical models for real-life data.

Teaching Tips

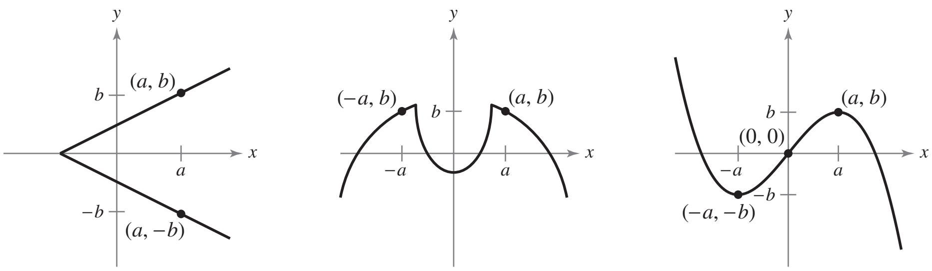
You may want to spend time reviewing factoring, solving equations involving square roots, and solving polynomial equations. For further review, encourage students to study the following material in *Precalculus*, 10th edition, by Larson.

- Factoring: Appendix A.3
- Solving equations involving square roots: Appendix A.5
- Solving polynomial equations: Appendix A.5

Encourage students who have access to a graphing utility or computer algebra system to use the technology to check their answers. Remind students that they should also be graphing using paper and pencil.

Start class by having students practice finding intercepts of the graph of an equation. Consider doing an in-class example of finding the intercepts of the graph of an equation with a radical, such as $y = \sqrt{x+4}$ or $y = x\sqrt{4-x^2}$.

Students have a hard time deciding if certain graphs are symmetric to the x-axis, y-axis, and origin. To help their understanding, tell students that the word *symmetric* conveys balance. If you want to set a dinner table, you want to have matching plates, utensils, and everything in line. To further aid students' understanding of symmetry, draw pictures of various symmetries. Suggested examples are shown below:

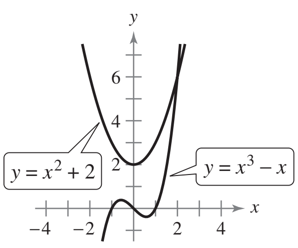


x-axis symmetry $\quad$ y-axis symmetry $\quad$ Origin symmetry

When finding points of intersection, it is useful to have students find the points both algebraically and by using a graphing utility. Use Example 5 on page 6 to find the points of intersection using a graphing utility.

How Do You See It? Exercise

Page 9, Exercise 74 Use the graphs of the two equations to answer the questions below.



(a) What are the intercepts for each equation?

(b) Determine the symmetry for each equation.

(c) Determine the point of intersection of the two equations.

Solution

(a) Intercepts for $y = x^3 - x$:

y-intercept: $y = 0^3 - 0 = 0$; $(0, 0)$

x-intercepts: $0 = x^3 - x = x(x^2 - 1) = x(x - 1)(x + 1)$; $(0, 0), (1, 0), (-1, 0)$

Intercepts for $y = x^2 + 2$:

y-intercept: $y = 0 + 2 = 2$; $(0, 2)$

x-intercepts: $0 = x^2 + 2$

None, y cannot equal 0.

(b) Symmetry with respect to the origin for $y = x^3 - x$ because

$-y = (-x)^3 - (-x) = -x^3 + x.$

Symmetry with respect to the y-axis for $y = x^2 + 2$ because

$y = (-x)^2 + 2 = x^2 + 2.$

(c)
$$x^3 - x = x^2 + 2$$
$$x^3 - x^2 - x - 2 = 0$$
$$(x - 2)(x^2 + x + 1) = 0$$
$$x = 2 \implies y = 6$$

Point of intersection: $(2, 6)$

Note: The polynomial $x^2 + x + 1$ has no real roots.

Suggested Homework Assignment

Pages 8–9: 1–15 odd, 19–55 odd, 65, and 67.

Section 1.2 Linear Models and Rates of Change

Section Comments

1.2 **Linear Models and Rates of Change**—Find the slope of a line passing through two points. Write the equation of a line with a given point and slope. Interpret slope as a ratio or as a rate in a real-life application. Sketch the graph of a linear equation in slope-intercept form. Write equations of lines that are parallel or perpendicular to a given line.

Teaching Tips

Spend time reviewing the following concepts: slope, writing equations of lines, and slope as a rate of change. For further review, encourage students to study the following material in *Precalculus*, 10th edition, by Larson.

- Slope: Section 1.3
- Finding the slope of a line: Section 1.3
- Writing linear equations in two variables: Section 1.3
- Parallel and perpendicular lines: Section 1.3
- Slope as a rate of change: Section 1.3

Encourage students who have access to a graphing utility or computer algebra system to use the technology to check their answers.

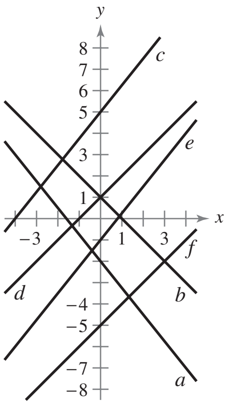
Review with the class that slope measures the steepness of a line. In addition, the slope of a line is a rate of change. Rate of change is an important topic in calculus, and inform students that rate of change will come up later in the semester.

Consider doing an example in class to remind students how to rewrite an equation such as $x + 3y = 12$ in slope-intercept form. Then show them how to identify the slope and y-intercept. Remind students that the slope of a vertical line is undefined and the slope of a horizontal line is 0. (As mentioned before, you can also direct students to the appropriate material in *Precalculus*.)

When finding equations of lines, consider writing each solution in four ways: slope-intercept form, two equations in point-slope form (depending on which point is chosen for (x_1, y_1)), and in general form. This way, students will see how to get from one form to the next.

How Do You See It? Exercise

Page 18, Exercise 72 Several lines are shown in the figure below. (The lines are labeled *a–f*.)



(a) Which lines have a positive slope?

(b) Which lines have a negative slope?

(c) Which lines appear parallel?

(d) Which lines appear perpendicular?

Solution

(a) Lines c, d, e, and f have positive slopes.

(b) Lines a and b have negative slopes.

(c) Lines c and e appear parallel. Lines d and f appear parallel.

(d) Lines b and f appear perpendicular. Lines b and d appear perpendicular.

Suggested Homework Assignment

Pages 16–18: 1, 11–23 odd, 27, 33, 41, 53, 61, 73, and 75.

Section 1.3 Functions and Their Graphs

Section Comments

1.3 **Functions and Their Graphs**—Use function notation to represent and evaluate a function. Find the domain and range of a function. Sketch the graph of a function. Identify different types of transformations of functions. Classify functions and recognize combinations of functions.

Teaching Tips

Spend time reviewing the following concepts: evaluating a function and the domain and range of a function. For further review, encourage students to study the following material in *Precalculus*, 10th edition, by Larson.

- Evaluating a function: Section 1.4
- Domain and range of a function: Section 1.4

Encourage students who have access to a graphing utility or computer algebra system to use the technology to check their answers.

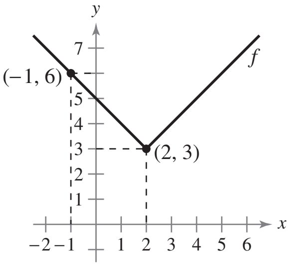
When evaluating a function, ask students what the function is doing with any value of x. For example, use the function:

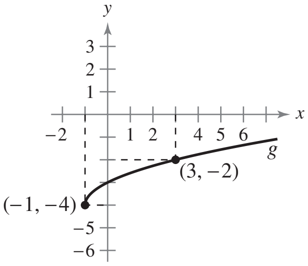
$f(x) = 2x + 3.$

Here, the function $f(x)$ takes a value of x, multiplies it by 2 and adds 3. Have students quickly fill out $f(1)$, $f(2)$, $f(-3)$, $f(0)$, $f(a)$, $f(b)$, and f(a student's name). Follow up with asking what $f(x)$ does with stuff, $f(\text{stuff}) = 2 \times \text{stuff} + 3$. This will prove to be useful for students when evaluating functions using the difference quotient. Then have students evaluate $f(4a)$ and $f(b + 2)$.

Be sure to spend some time evaluating functions using the difference quotient. Some suggested functions to use are: $f(x) = 2x^2 + 5x + 1$, $g(x) = \dfrac{1}{x - 3}$, and $h(x) = \sqrt{x + 2}$. Using $g(x)$ will test students' ability to find a least common denominator and using $h(x)$ will test students' ability to rationalize a numerator. Be sure to tell students that rationalizing the numerator will become useful in calculus.

When asking students to find the domain and range of functions, using different colored chalk or Expo markers help students visualize what the domains and ranges should be. For example, graph $f(x) = |x - 2| + 3$ and $g(x) = \sqrt{x + 1} - 4$. After graphing these two functions by using the transformations, tell students that if you pick any point on either f or g and map the point back to the x- and y-axes using different colors, students will be more likely to see what the domain and ranges are. Examples of these two functions are graphed below.





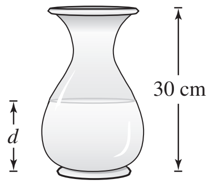
Remind students that the absolute value function is a piecewise function, $f(x) = |x| = \begin{cases} x, & x \ge 0 \\ -x, & x < 0 \end{cases}$. Discuss its domain and range, and if it is one-to-one. Point out the list of Basic Types of Transformations on page 23. Remind students that knowing the names of functions is important (pages 24–25), and that this can be useful in justifying later work.

When going over the Leading Coefficient Test, start by graphing the simplest monomial functions $f(x) = x^2$ and $g(x) = x^3$. Describe the end behavior of each. Next, insert negatives and describe the end behaviors. Summarizing the results, if the degree is even with a positive leading coefficient, both ends will be rising; if a negative leading coefficient, both sides will be falling. If the degree is odd with a positive leading coefficient, the right side will be rising and the left will be falling. The opposite is true if the leading coefficient is negative. Expose students to the notation $x \to \infty$ and $f(x) \to -\infty$ so that they will have a preview of calculus once limits are reached.

How Do You See It? Exercise

Page 30, Exercise 94

Water runs into a vase of height 30 centimeters at a constant rate. The vase is full after 5 seconds. Use this information and the shape of the vase shown to answer the questions when d is the depth of the water in centimeters and t is the time in seconds (see figure).



(a) Explain why d is a function of t.

(b) Determine the domain and range of the function.

(c) Sketch a possible graph of the function.

(d) Use the graph in part (c) to approximate $d(4)$. What does this represent?

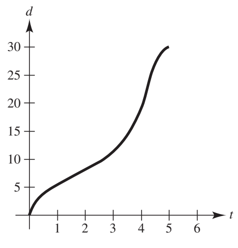
Solution

(a) For each time t, there corresponds a depth d.

(b) Domain: $0 \le t \le 5$

Range: $0 \le d \le 30$

(c)



(d) $d(4) \approx 18$. At time 4 seconds, the depth is approximately 18 centimeters.

Suggested Homework Assignment

Pages 27–30: 1, 3, 7, 11, 13–21 odd, 23, 27, 35, 37, 39, 41, 45, 49, 51–61 odd, 65, 67, 69, 75, 79, 85, and 103–107 odd.

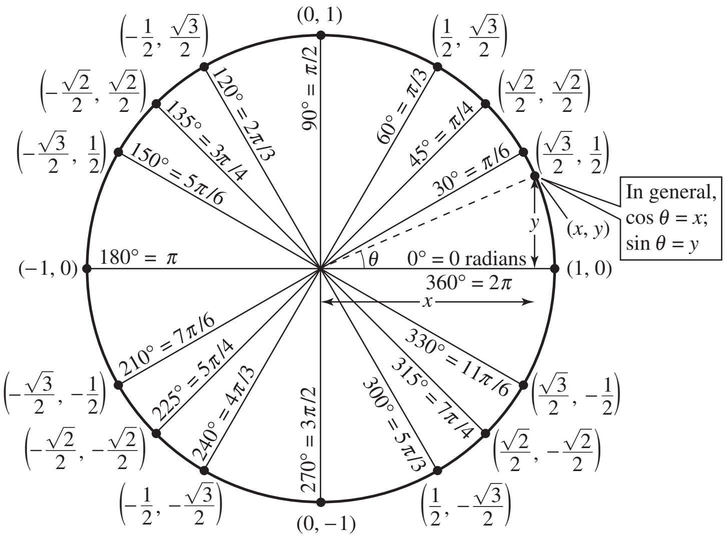
Section 1.4 Review of Trigonometric Functions

Section Comments

1.4 **Review of Trigonometric Functions**—Describe angles and use degree measure. Use radian measure. Understand the definitions of the six trigonometric functions. Evaluate trigonometric functions. Solve trigonometric equations. Graph trigonometric functions.

Teaching Tips

Start by asking students to recall the unit circle, the six trigonometric functions, and the signs for each quadrant. One can review SOHCAHTOA for right triangles. For the *xy*-plane, SYRCXRTYX can also be used as shown below.



SYRCXRTYX

$\sin\theta = \dfrac{y}{r}$

$\cos\theta = \dfrac{x}{r}$

$\tan\theta = \dfrac{y}{x}$

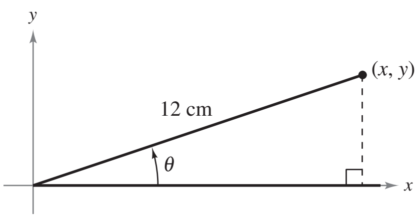
Quickly review problems on how to convert from radians to degrees and degrees to radians. Exercises 10 and 12 are good practice for class.

Students have a difficult time solving trigonometric equations, especially those that involve multiple angles. A suggested problem to work out with students is $3\cos x + 3 = 2\sin^2 x$, where $0 \le x \le 2\pi$. This will test students' memory on $\sin^2 x + \cos^2 x = 1$. Then to review multiple angles, work out the problem $\sin(2x) = \sin(x)$.

Lastly, problems using the power-reducing formulas should also be presented as they will be used for trigonometric integration. For example, simplify $\sin^4 x$.

How Do You See It? Exercise

Page 40, Exercise 72 Consider an angle in standard position with $r = 12$ centimeters, as shown in the figure. Describe the changes in the values of x, y, $\sin\theta$, $\cos\theta$, and $\tan\theta$ as θ increases continually from 0° to 90°.



Solution

As θ increases from $0°$ to $90°$ with $r = 12$ centimeters, x decreases from 12 to 0 centimeters, y increases from 0 to 12 centimeters, $\sin\theta$ increases from 0 to 1, $\cos\theta$ decreases from 1 to 0, and $\tan\theta$ increases from 0 to (positive) infinity.

Suggested Homework Assignment

Pages 38–40: 1–15 odd, 19, 23, 29, 31–51 odd, 55–65 odd, 69, 77, and 79.

Section 1.5 Inverse Functions

Section Comments

1.5 **Inverse Functions**—Verify that one function is the inverse function of another function. Determine whether a function has an inverse function. Develop properties of the six inverse trigonometric functions.

Teaching Tips

Students should be familiar with inverse functions from their algebra courses, so the first part of Section 1.5 could be covered quickly. However, they probably are not familiar with the idea of a one-to-one function, so spend some time explaining this idea and use the terminology frequently. Inverse trigonometric functions might not appeal to your students. Be sure to do problems like those in Example 7. The concept of using a right triangle is important for Chapters 5 and 8.

Consider doing an example with finding an inverse of a rational function such as: $f(x) = \dfrac{2x - 3}{5 + 7x}$.
Discuss how this function passes the Horizontal Line Test and review finding vertical and horizontal asymptotes.

Another suggested problem to cover with students is:
The point $\left(\dfrac{3\pi}{2}, 0\right)$ is on the graph of $y = \cos x$. Does $\left(0, \dfrac{3\pi}{2}\right)$ lie on the graph of $y = \arccos x$? If not, does this contradict the definition of inverse function?

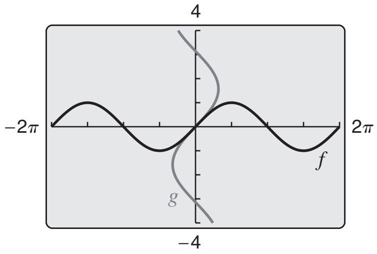
This example is a good review of the following concepts:

- Definition of an inverse function
- Domain and range of the cosine and arccosine functions
- Properties of inverse trigonometric functions

Use the graph of $y = \cos x$ and $y = \arccos x$. Note that the point $\left(\dfrac{3\pi}{2}, 0\right)$ lies on the graph of $y = \cos x$, but $\left(0, \dfrac{3\pi}{2}\right)$ does not lie on the graph of $y = \arccos x$. This does not contradict the definition of an inverse function because the domain of $y = \arccos x$ is $[-1, 1]$ and its range is $[0, \pi]$. So, $\arccos(0) = \dfrac{\pi}{2}$. Be sure to cover the material at the top of page 47 and the Properties of Inverse Trigonometric Functions.

How Do You See It? Exercise

Page 50, Exercise 90 You use a graphing utility to graph $f(x) = \sin x$ and then use the *draw inverse* feature to graph g (see figure). Is g the inverse function of f? Why or why not?



Solution

No, g is not the inverse of f. $f(x) = \sin x$ is not one-to-one. The graph of g is not the graph of a function.

Suggested Homework Assignment

Pages 48–51: 1, 5–9 odd, 13, 17, 19–43 odd, 45, 69–85 odd, 91–101 odd, 113–131 odd, and 139–143 odd.

Section 1.6 Exponential and Logarithmic Functions

Section Comments

1.6 **Exponential and Logarithmic Functions**—Develop and use properties of exponential functions. Understand the definition of the number e. Understand the definition of the natural logarithmic function, and develop and use properties of the natural logarithmic function, and develop and use properties of the natural logarithmic function.

Teaching Tips

Although students should be familiar and comfortable with both the exponential and logarithmic functions, sometimes they are not. Therefore, in Section 1.6, present these functions as though they are new ideas. Have your students commit to memory the graphs and properties of these functions. Your students will learn more about the number e in Chapter 2 when limits are discussed.

Use Example 3 to show students how to find the decimal approximation of e. You may want to use the following mnemonic device to help students remember the first six digits of e. "Andrew Jackson served two terms as the 7th President of the United States. He was elected to his first Presidency in 1828."

Remind students that the natural logarithmic function is the logarithmic function to the base e and is written as $\log_e x = \ln x$, where $x > 0$.

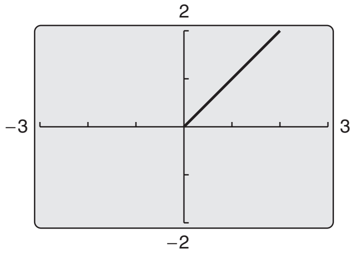
Try to show more examples of expanding logarithmic expressions than condensing, since students will need this skill when performing derivatives using logarithms.

You may want to reference Figure 1.62 on page 55 to help students describe the relationship between the graphs of $f(x) = e^x$ and $g(x) = \ln x$. Because the natural logarithmic function and the natural exponential function are inverses, the graphs of $f(x) = e^x$ and $g(x) = \ln x$ are mirror images across the line $y = x$.

How Do You See It? Exercise

Page 59, Exercise 128

The figure below shows the graph of $y_1 = \ln e^x$ or $y_2 = e^{\ln x}$. Which graph is it? What are the domains of y_1 and y_2? Does $\ln e^x = e^{\ln x}$ for all real values of x? Explain.



Solution

The graph is that of $y_2 = e^{\ln x}$.

The domain of $y_1 = \ln(e^x)$ is $(-\infty, \infty)$.

The domain of $y_2 = e^{\ln x}$ is $x > 0$.

No, $\ln e^x \neq e^{\ln x}$ for all real values of x. They are equal for $x > 0$.

Suggested Homework Assignment

Pages 57–59: 11–39 odd, 51, 77, 79, 89–117 odd, and 123.

Chapter 1 Project

Height of a Ferris Wheel Car

The Ferris wheel was designed by American engineer George Ferris (1859–1896). The first Ferris wheel was built for the 1893 World's Columbian Exposition in Chicago, and later used at the 1904 World's Fair in St. Louis. It had a diameter of 250 feet, and each of its 36 cars could hold 60 passengers.

Exercises

In Exercises 1–3, use the following information. A Ferris wheel with a diameter of 100 feet rotates at a constant rate of 4 revolutions per minute. Let the center of the Ferris wheel be at the origin.

1. Each of the Ferris wheel's cars travels around a circle.

(a) Write an equation of the circle, where x and y are measured in feet.

(b) Sketch a graph of the equation you wrote in part (a).

(c) Use the Vertical Line Test to determine whether y is a function of x.

(d) What does your answer to part (c) mean in the context of the problem?

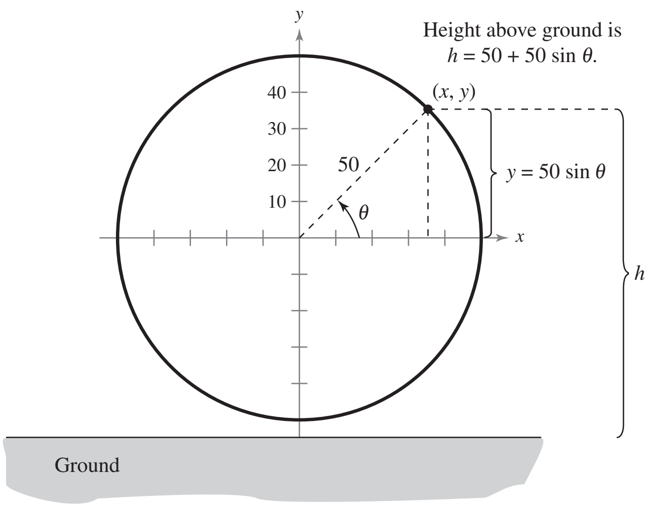
2. The height h (in feet) of a Ferris wheel car located at the point (x, y) is given by

$h = 50 + y$

where y is related to the angle θ (in radians) by the equation

$y = 50 \sin \theta$

as shown in the figure.



(a) Write an equation of the height h in terms of time t (in minutes). (*Hint:* One revolution is 2π radians.)

(b) Sketch a graph of the equation you wrote in part (a).

(c) Use the Vertical Line Test to determine whether h is a function of t.

(d) What does your answer to part (c) mean in the context of the problem?

3. The model in Exercise 2 yields a height of 50 feet when $t = 0$. Alter the model so that the height of the car is 0 feet when $t = 0$. Explain your reasoning.

